Eur. Phys. J. C 31, 177–185 (2003)
Digital Object Identifier (DOI) 10.1140/epjc/s2003-01338-6

THE EUROPEAN PHYSICAL JOURNAL C

Algebraic solution of RPA equations for CC quasi-elastic neutrino-nucleus scattering

K.M. Graczyk[a], J.T. Sobczyk[b]

Institute of Theoretical Physics, University of Wrocław, pl. M. Borna 9, 50-204, Wrocław, Poland

Received: 25 March 2003 / Revised version: 26 August 2003 /
Published online: 10 October 2003 –

Abstract. An algebraic solution of the RPA equations for nucleon re-interactions in the case of quasi-elastic charged-current neutrino–nucleus scattering is presented. The Abelian algebra of the matrices allows us to extract four independent corrections to the cross section separately. The results of numerical computations are shown.

1 Introduction

A better theoretical understanding of nuclear effects in neutrino–nucleus scattering is important in view of the data analysis from new more precise neutrino experiments such as K2K, MINOS, MiniBoone. The determination of the parameters governing the neutrino oscillation phenomenon, in particular of θ_{13}, requires improved knowledge of the neutrino–nucleus cross sections [1]. In the above mentioned experiments the neutrino beam energy is of the order of 1 GeV. A characteristic feature of neutrino–nucleus reactions in this energy domain is the formation of resonances and subsequent pion production. However, the quasi-elastic contribution is still important and its precise determination is of interest.

Nuclear effects in the neutrino–nucleus interaction are often evaluated in the framework of a Monte Carlo approach. Scattering is split into two steps. The neutrino interacts first with a free nucleon; outgoing particles are then subject to re-interactions inside the nucleus. In more systematic theoretical approaches mean field theory with a relativistic Fermi gas of protons and neutrons as a ground state can serve as one of the techniques to provide a model for the nucleus [2]. In its simplest version the nucleus forms a sea of fermions with momentum uniformly distributed inside the Fermi sphere. The effect of MFT is that one has to substitute nucleon mass M by an effective mass M^*. It is well known that for energies in the GeV range in the case of the electron–nucleus scattering Fermi gas model with fine tuned values of the Fermi momentum and effective mass accounts for basic features of the dynamics [3]. More realistically, nucleons interact with each other exchanging pions and ρ mesons, and also short range correlations have to be considered by introducing suitable contact interactions terms [4]. In the ring approximation of the RPA approach a summation over all Feynman diagrams is substituted by a sum of diagrams where only 1p-1h (one particle–one hole) excitations are included [5]. In order to make the theory better one should also consider elementary 2p-2h excitations in order to enlarge the cross section in the so-called "dip" region [6, 7]. It is however difficult to include this contribution in the RPA scheme [8].

In this paper analytic expressions for four contributions to RPA corrections are derived in the case of quasi-elastic neutrino reactions. An algebra of matrices is introduced to solve the Dyson equation. Results for separate contributions are presented. It is known [9] that RPA corrections typically reduce the maximum in the energy transfer differential cross section by a factor of about 10%. Our formalism when applied to the CC process is only expected to reproduce these results. In particular we want to mention here the paper [10]. We try to keep the same notation in order to make a comparison easier. A first motivation for the present study is to construct a general framework in which a more detailed analysis of quasi-elastic CC processes could be possible. One can evaluate the significance of uncertainties in various parameters: nucleons' form factors, coupling constants, effective mass etc. The second motivation is that the same algebraic framework can be applied to NC reactions and hopefully also (with necessary modifications) to the Δ excitation. In our RPA computations we keep a constant value for the Fermi momentum. Also M^* is assumed to be a function of Fermi momentum only; thus it is a constant. Inclusion of local density effects in the analytical framework is in principle possible but rather complicated – a lot of manipulations with spherical harmonics are necessary [7] with approximations difficult to control. For all practical purposes it is sufficient to have an exact cross section formula for a fixed value of Fermi momentum (and M^*) since one can perform a numerical integration over Fermi momenta with a distribution defined by the density profile of the nucleus in question. Our

[a] e-mail: kgraczyk@ift.uni.wroc.pl
[b] e-mail: jsobczyk@ift.uni.wroc.pl

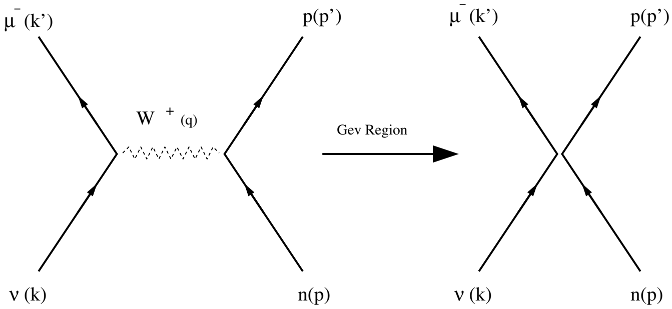

Fig. 1. The basic diagram describing neutrino–nucleus interaction. Scattering takes place on a single nucleon with a definite momentum given by the Fermi gas distribution. In the energy domain of a few GeV the effective four-Fermion vertex provides an excellent approximation

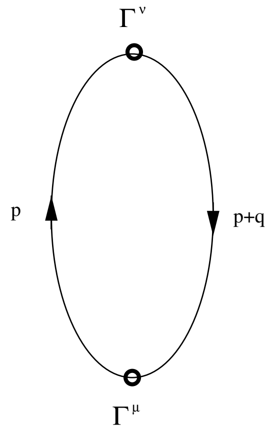

Fig. 2. $\Pi^{\mu\nu}_{\mathrm{free}}$

algebraic scheme enables a rather simple computation of both effects (RPA and local density) and this is the third motivation for our work. We wish to mention that a similar idea for solving the Dyson equation can be found in [11].

This paper is organized as follows. A short description of the model, Feynman rules and the main technical tool - algebra of matrices is given in Sect. 2. Explicit expressions for RPA corrections are presented in Sect. 3. Section 4 contains a discussion of results and also a comparison with RPA computations done in the relativistic generalization of the Marteau model [12]. There is a very nice agreement between the two approaches. Some technical details of algebraic computations are collected in the appendix. The aim of this paper is to present the main features of this approach. More details and discussion will be in our next paper, which is now in preparation.

2 The formalism

We consider charged-current (CC) quasi-elastic neutrino–nucleus scattering (Fig. 1). The model of the nucleus is given by the mean field theory [2] (the nucleon mass becomes an effective mass $M^*(k_{\mathrm{F}})$) with interactions due to contact terms and exchange of pions and ρ mesons [4]. In a first approximation the nucleus is treated as a relativistic Fermi gas with Fermi momentum k_{F} determined by the nucleons' density.

The elementary weak charged-current nucleon–nucleon current is expressed by means of form factors [13]:

$$\Gamma^{\alpha}(q_\mu) = F_1(q_\mu^2)\gamma^\alpha + F_2(q_\mu^2)\frac{\mathrm{i}\sigma^{\alpha\nu}q_\nu}{2M} + G_{\mathrm{A}}(q_\mu^2)\gamma^\alpha\gamma^5. \quad (1)$$

We omit the $G_p(q_\mu)$ term since its contribution to the ν_e and ν_μ cross sections at $E_\nu \sim 1\,\mathrm{GeV}$ is negligible. It can be put into our scheme if required with only minor modifications.

The differential cross section (per nucleon) reads

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}|\mathbf{q}|\,\mathrm{d}q_0} = -\frac{G_{\mathrm{F}}^2\cos^2\theta_c\,|\mathbf{q}|}{16\pi^2\rho_{\mathrm{F}}E^2}\mathrm{Im}\left(L_\mu{}^\nu\Pi^\mu{}_\nu\right). \quad (2)$$

$\rho_{\mathrm{F}} = k_{\mathrm{F}}^3/3\pi^2$, and we assume the same values of the Fermi momentum for neutrons and protons. $L^{\mu\nu}$ is the leptonic tensor:

$$L_{\mu\nu} = 8\left(k_\mu k'_\nu + k'_\mu k_\nu - g_{\mu\nu}k_\alpha k'^\alpha \pm \mathrm{i}\epsilon_{\mu\nu\alpha\beta}k'^\alpha k^\beta\right). \quad (3)$$

The sign $\pm$ depends on the process considered (neutrino/antineutrino).

The polarization tensor $\Pi^{\mu\nu}$ is a basic object that contains full information about nuclear effects. It is defined as a chronological product of many body currents:

$$\Pi^{\mu\nu}(q_0, q) = -\mathrm{i}\int \mathrm{d}^4x\,\mathrm{e}^{\mathrm{i}q_\alpha x^\alpha}\langle 0|T\left(\mathcal{J}^\mu(x)\mathcal{J}^\nu(0)\right)|0\rangle. \quad (4)$$

The $|0\rangle$ is the ground state of the nucleus described by the Fermi gas model. The prescription for nuclear physics [14] enables the evaluation of $\Pi^{\mu\nu}$ by means of the standard QFT techniques with modified (depending on k_{F}) progagator $G(p)$.

2.1 Feynman rules

The polarization tensor is split into a "free" part and a RPA correction:

$$\Pi^{\mu\nu} = \Pi^{\mu\nu}_{\mathrm{free}} + \Delta\Pi^{\mu\nu}_{\mathrm{RPA}}. \quad (5)$$

The "free" tensor is given by a simple fermion loop (Fig. 2) which is spanned between two vertices with form factor insertions.

$$\Pi^{\mu\nu}_{\mathrm{free}}(q) \quad (6)$$
$$= -\mathrm{i}\int\frac{\mathrm{d}^4p}{(2\pi)^4}\mathrm{Tr}\left(G(p+q)\Gamma^\mu(q)G(p)\Gamma^\nu(-q)\right),$$
$$G(p) = (\not{p} + M^*) \quad (7)$$
$$\times\left(\frac{1}{p_\alpha^2 - M^{*2} + \mathrm{i}\epsilon} + \frac{\mathrm{i}\pi}{E_p}\delta(p_0 - E_p)\theta(k_{\mathrm{F}} - p)\right).$$

$G(p)$ describes the propagation of a free fermion in the Fermi sea.

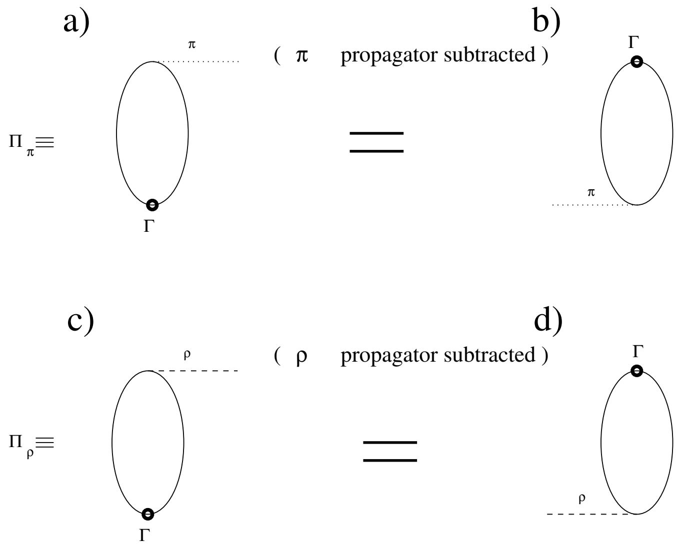

Fig. 3a–d. Diagrammatical definitions of $\Pi_{\pi,\rho}$. They are loops with CC weak nucleon–nucleon current in one vertex and $NN\pi$ or $NN\rho$ vertices

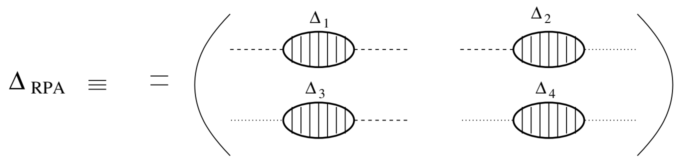

Fig. 4. Following [10] we define the RPA propagator as a sum over 1p-1h diagrams with external meson fields propagators

RPA corrections arise from the summation of an infinite sum of 1p-1h contributions [5]. These corrections are given by Feynman diagrams containing the π propagator $V^{\mu\nu}$, the ρ propagator $W^{\mu\nu}$ and interaction vertices $NN\pi$ as well as $NN\rho$. The Landau–Migdal parameter g' is put together with a genuine pion propagator to form a redefined pion "propagator" (for details see [10]).

In what follows in this section we do not write down explicitly Lorentz indices. It will be understood that unless specified all the objects are 4×4 matrices with indices $(\ )_\mu{}^\nu$. We define the tensors Π_ρ and Π_π as loop diagrams with a CC weak nucleon–nucleon current in one vertex and $NN\rho$ or $NN\pi$ vertices (Fig. 3). Notice that contributions from tensors given by Figs. 3a,b are equal. The same applies to the contributions of Fig. 3c,d. This property simplifies the algebraic form of the RPA corrections.

$\Delta_{\rm RPA}$ is an 8×8 matrix defined by an infinite series:

$$\Delta_{\rm RPA} = \Delta_0 + \Delta_0 \Pi_G \Delta_0 + \Delta_0 \Pi_G \Delta_0 \Pi_G \Delta_0 + ..., \qquad (8)$$

which is illustrated in Fig. 5. We defined two new tensors, the 8×8 matrices Π_G and Δ_0. Their definitions can be simply understood in diagrammatical language (Fig. 6).

With all these definitions the formula (8) can be rewritten in the form of the Dyson equation (Fig. 7):

$$\Delta_{\rm RPA} = \Delta_0 + \Delta_0 \Pi_G \Delta_{\rm RPA}. \qquad (9)$$

It is clear that RPA corrections to the polarization propagator are given by $\Delta_{\rm RPA}$ multiplied from both sides

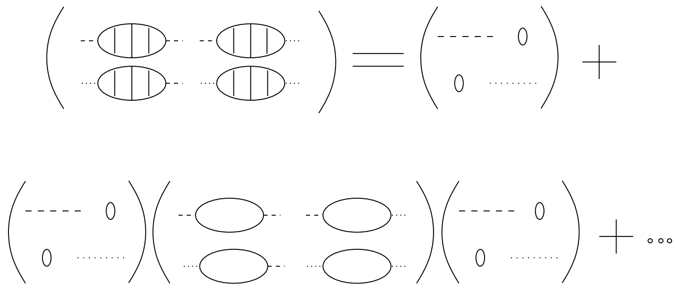

Fig. 5. The propagator RPA can be expressed as an infinite sum

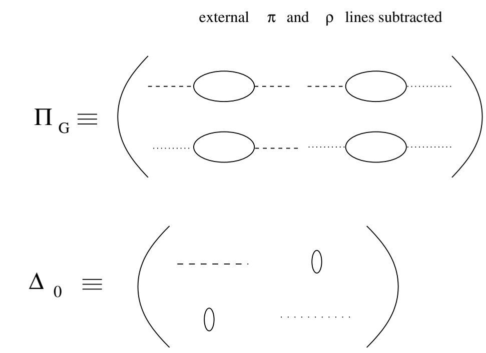

Fig. 6. The tensors Π_G and D_0

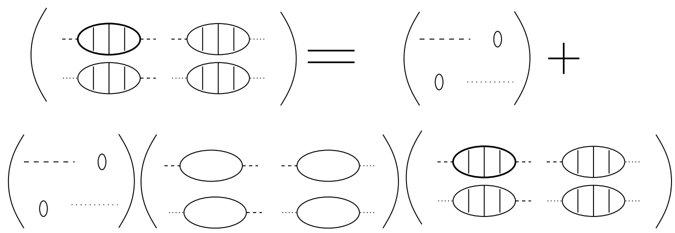

Fig. 7. Dyson equation for the RPA propagator

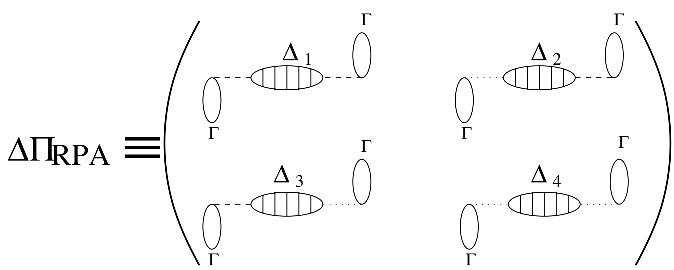

Fig. 8. Diagrammatical explanation of RPA corrections to the polarization propagator

by Π_ρ and Π_π;

$$\Delta\Pi_{\rm RPA}(q_\mu) = \begin{pmatrix} \Pi_\rho(q_\mu) \\ \Pi_\pi(q_\mu) \end{pmatrix} \Delta_{\rm RPA}(q_\mu) \begin{pmatrix} \Pi_\rho(q_\mu) \\ \Pi_\pi(q_\mu) \end{pmatrix}. \qquad (10)$$

The diagrammatic explanation of this formula is presented in Fig. 8. Our strategy is to solve the equation for $\Delta_{\rm RPA}$ and then to obtain the expression for $\Delta\Pi_{\rm RPA}$.

2.2 Algebraic properties of the polarization tensor

A coordinate system is chosen in which the four momentum transfer reads

$$q_\mu = k_\mu - {k'}_\mu = (q_0, q, 0, 0). \tag{11}$$

The Dyson equation is a 8×8 matrix equation. In order to solve it we introduce 4×4 matrices:

$$e_\mathrm{L} = \begin{pmatrix} -\frac{q^2}{q_\mu^2} & \frac{q_0 q}{q_\mu^2} & 0 & 0 \\ -\frac{q_0 q}{q_\mu^2} & \frac{q_0^2}{q_\mu^2} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad e_\mathrm{T} = \begin{pmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix},$$

$$e_\mathrm{A} = \begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix}, \qquad e_\mathrm{VA} = \begin{pmatrix} 0&0&0&0 \\ 0&0&0&0 \\ 0&0&0&-\mathrm{i} \\ 0&0&\mathrm{i}&0 \end{pmatrix}. \tag{12}$$

They satisfy the matrix multiplication relations

$$\begin{aligned} e_\mathrm{L}\, e_\mathrm{L} &= e_\mathrm{L}, \\ e_\mathrm{L}\, e_\mathrm{T} &= 0 = e_\mathrm{T}\, e_\mathrm{L}, \\ e_\mathrm{L}\, e_\mathrm{A} &= e_\mathrm{L} = e_\mathrm{A}\, e_\mathrm{L}, \\ e_\mathrm{L}\, e_\mathrm{VA} &= 0 = e_\mathrm{VA}\, e_\mathrm{L}, \\ e_\mathrm{T}\, e_\mathrm{T} &= e_\mathrm{T}, \\ e_\mathrm{T}\, e_\mathrm{A} &= e_\mathrm{T} = e_\mathrm{A}\, e_\mathrm{T}, \\ e_\mathrm{T}\, e_\mathrm{VA} &= e_\mathrm{VA} = e_\mathrm{VA}\, e_\mathrm{T}, \\ e_\mathrm{A}\, e_\mathrm{A} &= e_\mathrm{A}, \\ e_\mathrm{A}\, e_\mathrm{VA} &= e_\mathrm{VA} = e_\mathrm{VA}\, e_\mathrm{A}, \\ e_\mathrm{VA}\, e_\mathrm{VA} &= e_\mathrm{T}. \end{aligned} \tag{13}$$

The polarization tensor must be a linear combination of e_L, e_T, e_A, e_VA,

$$\Pi = \Pi^\mathrm{L} e_\mathrm{L} + \Pi^\mathrm{T} e_\mathrm{T} + \Pi^\mathrm{VA} e_\mathrm{VA} + \Pi^\mathrm{A} e_\mathrm{A}. \tag{14}$$

This is clear since Π is given by a sum over Feynman diagrams and each contribution is of this form due to the fact that the algebra $e_\mathrm{L}, e_\mathrm{T}, e_\mathrm{VA}, e_\mathrm{A}$ is closed under multiplication and all the building blocks are expressed in terms of these four basic matrices. Consequently the cross section has the form

$$\begin{aligned} \frac{\mathrm{d}^2\sigma}{\mathrm{d}q\,\mathrm{d}q_0} &= -\frac{G_\mathrm{F}^2 \cos^2\theta_c\, q}{16\pi^2 \rho_\mathrm{F} E^2} \\ &\times \mathrm{Im}\left(L_\mathrm{L}\Pi^\mathrm{L} + L_\mathrm{T}\Pi^\mathrm{T} \pm L_\mathrm{VA}\Pi^\mathrm{VA} + L_\mathrm{A}\Pi^\mathrm{A}\right), \end{aligned} \tag{15}$$

where $L_\mathrm{L} \equiv {L_\mu}^\nu {e_\mathrm{L}}^\mu{}_\nu$ etc. Many authors, e.g. [10] call the four contributions longitudinal, transverse, V-A, and axial. This can cause some confusion because contributions to the cross section are sometimes called after the spin–isospin operators present in the nucleon–nucleon transition current. We will return to this point in Sect. 4.

3 RPA corrections

A lot of simplifications come from the fact that Δ_0 contains only longitudinal, transverse and axial terms [10]. We have

$$\begin{aligned} \Delta_0 &= \begin{pmatrix} W & 0 \\ 0 & V \end{pmatrix} \\ &= \begin{pmatrix} W^\mathrm{L} e_\mathrm{L} + W^\mathrm{T} e_\mathrm{T} + W^\mathrm{A} e_\mathrm{A} & 0 \\ 0 & V^\mathrm{L} e_\mathrm{L} + V^\mathrm{T} e_\mathrm{T} + V^\mathrm{A} e_\mathrm{A} \end{pmatrix}. \end{aligned} \tag{16}$$

W and V are

$$W^\mathrm{L} = W^\mathrm{T} = -\frac{q_\mu^2/m_\rho^2}{q_\mu^2 - m_\rho^2 + \mathrm{i}\epsilon}, \tag{17}$$

$$W^\mathrm{A} = \frac{q_\mu^2 - m_\rho^2}{m_\rho^2} \frac{1}{q_\mu^2 - m_\rho^2 + \mathrm{i}\epsilon}, \tag{18}$$

$$V^\mathrm{L} = V^\mathrm{T} = -\frac{q_\mu^2}{q_\mu^2 - m_\pi^2 + \mathrm{i}\epsilon}, \tag{19}$$

$$V^\mathrm{A} = \frac{q_\mu^2}{q_\mu^2 - m_\pi^2 + \mathrm{i}\epsilon} - g'. \tag{20}$$

g' is the Landau–Migdal parameter.

Also we have

$$\Pi_\rho = \Pi_\rho^\mathrm{L} e_\mathrm{L} + \Pi_\rho^\mathrm{T} e_\mathrm{T} + \Pi_\rho^\mathrm{VA} e_\mathrm{VA}, \tag{21}$$

$$\Pi_\pi = \Pi_\pi^\mathrm{L} e_\mathrm{L} + \Pi_\pi^\mathrm{T} e_\mathrm{T} + \Pi_\pi^\mathrm{VA} e_\mathrm{VA} + \Pi_\pi^\mathrm{A} e_\mathrm{A}. \tag{22}$$

We find

$$\begin{aligned} \Pi_G &= \begin{pmatrix} \Pi_{\rho\rho} & \Pi_{\rho\pi} \\ \Pi_{\pi\rho} & \Pi_{\pi\pi} \end{pmatrix} \\ &= \begin{pmatrix} \Pi_{\rho\rho}^\mathrm{L} e_\mathrm{L} + \Pi_{\rho\rho}^\mathrm{T} e_\mathrm{T} & \Pi_{\rho\pi}^\mathrm{VA} e_\mathrm{VA} \\ \Pi_{\rho\pi}^\mathrm{VA} e_\mathrm{VA} & \Pi_{\pi\pi}^\mathrm{L} e_\mathrm{L} + \Pi_{\rho\rho}^\mathrm{T} e_\mathrm{T} + \Pi_{\rho\rho}^\mathrm{A} e_\mathrm{A} \end{pmatrix}. \end{aligned} \tag{23}$$

We introduce the general notation

$$\begin{aligned} \Delta_\mathrm{RPA} &= \begin{pmatrix} \Delta_1 & \Delta_2 \\ \Delta_3 & \Delta_4 \end{pmatrix}, \\ \Delta_i &= \Delta_i^\mathrm{L} e_\mathrm{L} + \Delta_i^\mathrm{T} e_\mathrm{T} + \Delta_i^\mathrm{VA} e_\mathrm{VA} + \Delta_i^\mathrm{A} e_\mathrm{A}, \\ &\quad i = 1,2,3,4. \end{aligned} \tag{24}$$

The 8×8 matrix equation (9) can be rewritten as a set of four 4×4 matrix equations:

$$\begin{aligned} \Delta_1 &= W + W\Pi_{\rho\rho}\Delta_1 + W\Pi_{\rho\pi}\Delta_3, \\ \Delta_2 &= W\Pi_{\rho\rho}\Delta_2 + W\Pi_{\rho\pi}\Delta_4, \\ \Delta_3 &= V\Pi_{\rho\pi}\Delta_1 + V\Pi_{\pi\pi}\Delta_3, \\ \Delta_4 &= V + V\Pi_{\rho\pi}\Delta_2 + V\Pi_{\pi\pi}\Delta_4. \end{aligned} \tag{25}$$

Equations (25) are transformed to a set of algebraic equations and solved in the appendix. We obtain a general solution for Δ_{RPA}:

$$\Delta_{\mathrm{RPA}} = \tag{26}$$

$$\begin{pmatrix} \Delta_1^{\mathrm{A}} e_{\mathrm{A}} + \Delta_1^{\mathrm{L}} e_{\mathrm{L}} + \Delta_1^{\mathrm{T}} e_{\mathrm{T}} & \Delta_2^{\mathrm{VA}} e_{\mathrm{VA}} \\ \Delta_3^{\mathrm{VA}} e_{\mathrm{VA}} & \Delta_4^{\mathrm{A}} e_{\mathrm{A}} + \Delta_4^{\mathrm{L}} e_{\mathrm{L}} + \Delta_4^{\mathrm{T}} e_{\mathrm{T}} \end{pmatrix}.$$

RPA corrections to the polarization tensor are

$$\Delta\Pi^{\mathrm{L}}_{\mathrm{RPA}} = \left(\left(\Pi^{\mathrm{L}}_{\pi}\right)^2 + 2\Pi^{\mathrm{L}}_{\pi}\Pi^{\mathrm{A}}_{\pi}\right)\left(\Delta_4^{\mathrm{A}} + \Delta_4^{\mathrm{L}}\right) + \left(\Pi^{\mathrm{A}}_{\pi}\right)^2 \Delta_4^{\mathrm{L}} + \left(\Pi^{\mathrm{L}}_{\rho}\right)^2 \left(\Delta_1^{\mathrm{A}} + \Delta_1^{\mathrm{L}}\right), \tag{27}$$

$$\begin{aligned}\Delta\Pi^{\mathrm{T}}_{\mathrm{RPA}} = {} & \left(\left(\Pi^{\mathrm{T}}_{\pi}\right)^2 + 2\Pi^{\mathrm{T}}_{\pi}\Pi^{\mathrm{A}}_{\pi} + \left(\Pi^{\mathrm{VA}}_{\pi}\right)^2\right)\left(\Delta_4^{\mathrm{A}} + \Delta_4^{\mathrm{T}}\right) \\ & + \left(\Pi^{\mathrm{A}}_{\pi}\right)^2 \Delta_4^{\mathrm{T}} \\ & + \left(\left(\Pi^{\mathrm{T}}_{\rho}\right)^2 + \left(\Pi^{\mathrm{VA}}_{\rho}\right)^2\right)\left(\Delta_1^{\mathrm{T}} + \Delta_1^{\mathrm{A}}\right) \\ & + \left(\Pi^{\mathrm{VA}}_{\rho}\Pi^{\mathrm{T}}_{\pi} + \Pi^{\mathrm{VA}}_{\rho}\Pi^{\mathrm{A}}_{\pi} + \Pi^{\mathrm{T}}_{\rho}\Pi^{\mathrm{VA}}_{\pi}\right) \\ & \times \left(\Delta_2^{\mathrm{VA}} + \Delta_3^{\mathrm{VA}}\right),\end{aligned} \tag{28}$$

$$\begin{aligned}\Delta\Pi^{\mathrm{VA}}_{\mathrm{RPA}} = {} & \left(\Pi^{\mathrm{T}}_{\rho}\Pi^{\mathrm{T}}_{\pi} + \Pi^{\mathrm{T}}_{\rho}\Pi^{\mathrm{A}}_{\pi} + \Pi^{\mathrm{VA}}_{\rho}\Pi^{\mathrm{VA}}_{\pi}\right)\left(\Delta_2^{\mathrm{VA}} + \Delta_3^{\mathrm{VA}}\right) \\ & +2\left(\Delta_1^{\mathrm{T}} + \Delta_1^{\mathrm{A}}\right)\Pi^{\mathrm{VA}}_{\rho}\Pi^{\mathrm{T}}_{\rho} \\ & +2\left(\Delta_4^{\mathrm{T}} + \Delta_4^{\mathrm{A}}\right)\left(\Pi^{\mathrm{A}}_{\pi} + \Pi^{\mathrm{T}}_{\pi}\right)\Pi^{\mathrm{VA}}_{\pi},\end{aligned} \tag{29}$$

$$\Delta\Pi^{\mathrm{A}}_{\mathrm{RPA}} = \left(\Pi^{\mathrm{A}}_{\pi}\right)^2 \Delta_4^{\mathrm{A}}. \tag{30}$$

Expressions for Δ_1^{A}... are written in the appendix.

4 Discussion

The numerical evaluation of RPA corrections to $\Pi^{\mu\nu}$ requires knowledge of both real and imaginary parts of all objects presented in (27)–(30). We get the necessary formulae from [15]. With this input one can perform a numerical analysis of our algebraic results.

We present some plots demonstrating the consistency of our procedures. We compare the RPA corrections obtained in our paper with those computed in a different approach. The approach we choose is based on the Marteau model with some improvements [12]. The use of a relativistic generalization of the Lindhard function made the kinematical regions of both models the same.

In the Marteau model three contributions to the cross section are identified according to the spin–isospin operators present in the transition amplitude. To find a bridge between two decompositions we observe that the hadronic tensor components $H^{00}, H^{01}, H^{10}, H^{11}$ contribute only to longitudinal and charge contributions (in the spin–isospin nomenclature) while the remaining components contribute only to the transverse part. This is strictly speaking true in the approximation when the $\frac{|\mathbf{p}|}{M}$ ($\mathbf{p}$ is the target's nucleon momentum) terms are neglected in $H^{\mu\nu}$ which is valid within a few %. We decided therefore to single out two contributions in both approaches and to call them in order to avoid confusion: I and II. Contribution II is equivalent to the sum of charge and longitudinal parts in the Marteau approach, while contribution I is equivalent to the transverse part.

The identification of the I and II parts in our approach requires some algebra. We obtain

$$\left(L_{\mu\nu}\Pi^{\mu\nu}\right)_{\mathrm{I}} = L_{\mathrm{T}}\left(\Pi^{\mathrm{T}} - \Pi^{\mathrm{A}}\right) + L_{\mathrm{VA}}\Pi^{\mathrm{VA}}, \tag{31}$$

$$\left(L_{\mu\nu}\Pi^{\mu\nu}\right)_{\mathrm{II}} = L_{\mathrm{L}}\Pi^{\mathrm{L}} + \left(L_{\mathrm{A}} + L_{\mathrm{T}}\right)\Pi^{\mathrm{A}}. \tag{32}$$

In our numerical calculations we assumed the following values of parameters present in the theory: $M_{\mathrm{A}} = 1.03\,\mathrm{GeV}$, the axial mass, standard values of the coupling constants for pions and ρ mesons, the Landau–Migdal parameter $g' = 0.7$ except for two comparison plots (in [12] the value $g' = 0.6$ was assumed and we take the same value in order to make the comparison consistent). The effective mass was calculated according to the self-consistency equation of MFT theory [2]. It is assumed that the target nucleus is oxygen $^{16}\mathrm{O}$ and that the Fermi momentum is $k_{\mathrm{F}} = 225\,\mathrm{MeV}$. We get $M^* = 638\,\mathrm{MeV}$.

In the Fig. 9 we compare predictions for the total cross section in three cases:
(i) free Fermi gas with $M^* = 939\,\mathrm{MeV}$,
(ii) RPA computations with $M^* = 939\,\mathrm{MeV}$, and
(iii) RPA computations with $M^* = 638\,\mathrm{MeV}$. The inclusion of RPA correlations makes the cross section smaller. In the third case the reduction of the cross section is more significant for neutrino energies up to about 3 GeV.

In Figs. 10 and 11 we show the differential cross sections in energy transfer for a neutrino energy of 1 GeV. As above we distinguish two cases in which the effective mass is taken either as the free mass of a nucleon or as 638 MeV. One can see the typical expected behavior: in the RPA case the quasi-elastic peak becomes reduced, but at larger values of the energy transfer the effect of RPA is to slightly increase the cross section. We notice that due to the effective mass the kinematically allowed regions in the energy transfer are in the two cases different.

In the last two figures we compare our differential cross sections with predictions of the model described in [12]. A good agreement between the influence of RPA corrections in the two models is seen. The contribution I is dominant in both cases. Differences between them are small. The behavior of contribution II in both cases is similar. The Marteau model gives rise to smaller contributions at an energy transfer of $\sim 50\,\mathrm{MeV}$ and the whole contribution becomes reduced by about 25%.

We conclude that our algebraic solution of RPA equations leads to modifications of the cross section similar to other approaches. We cannot expect that the Marteau model [12] can produce numerically identical results as it is a hybrid model which combines a non-relativistic potential approach with a relativistic Lindhard function. We hope that our algebraic scheme will be useful in other cases mentioned in the introduction.

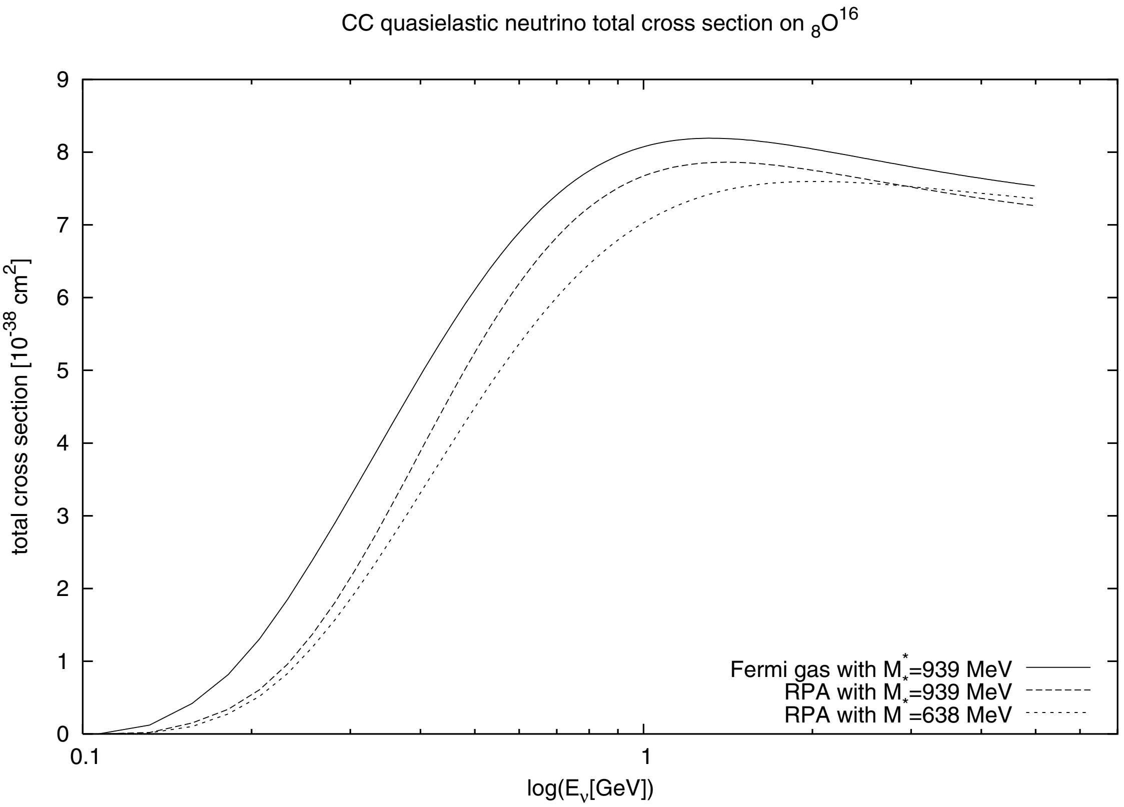

Fig. 9. Comparison of the computations of quasi-elastic neutrino total cross sections on oxygen and $g' = 0.7$

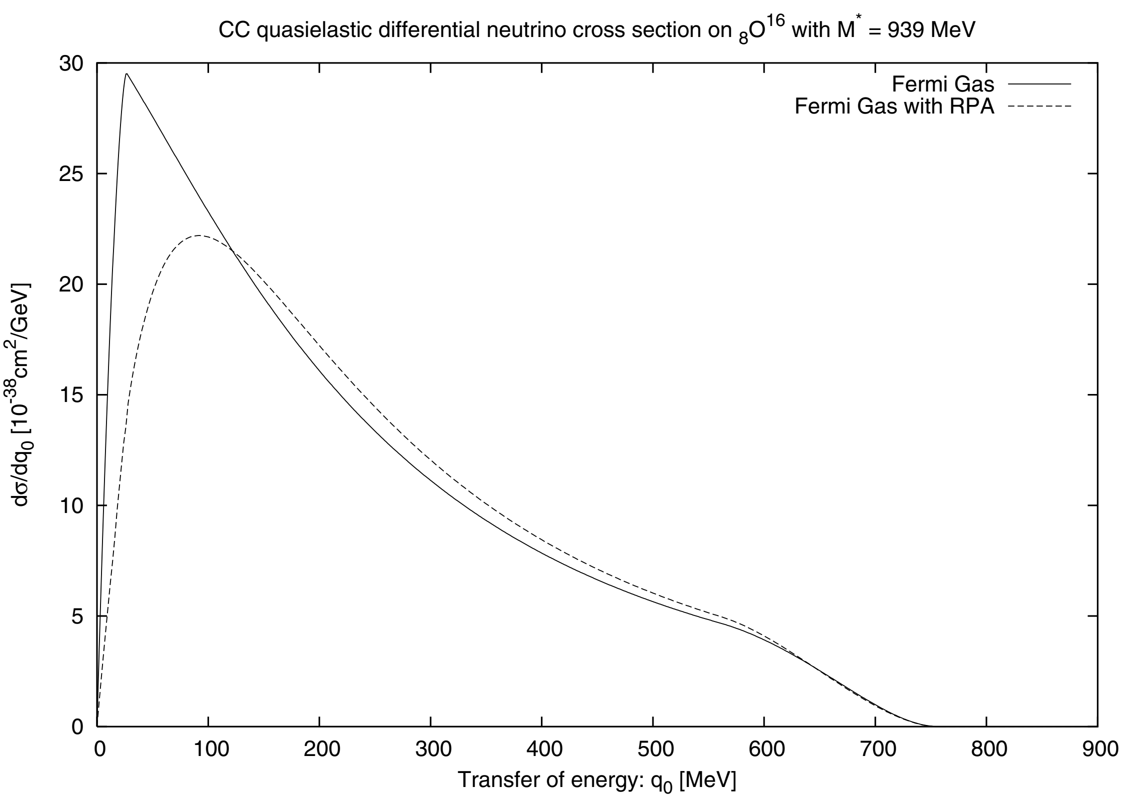

Fig. 10. Comparison of differential quasi-elastic neutrino cross section for a Fermi gas of free nucleons with cross section modified by RPA corrections for a neutrino energy of 1 GeV. The calculation was done for the free nucleon mass and $g' = 0.7$

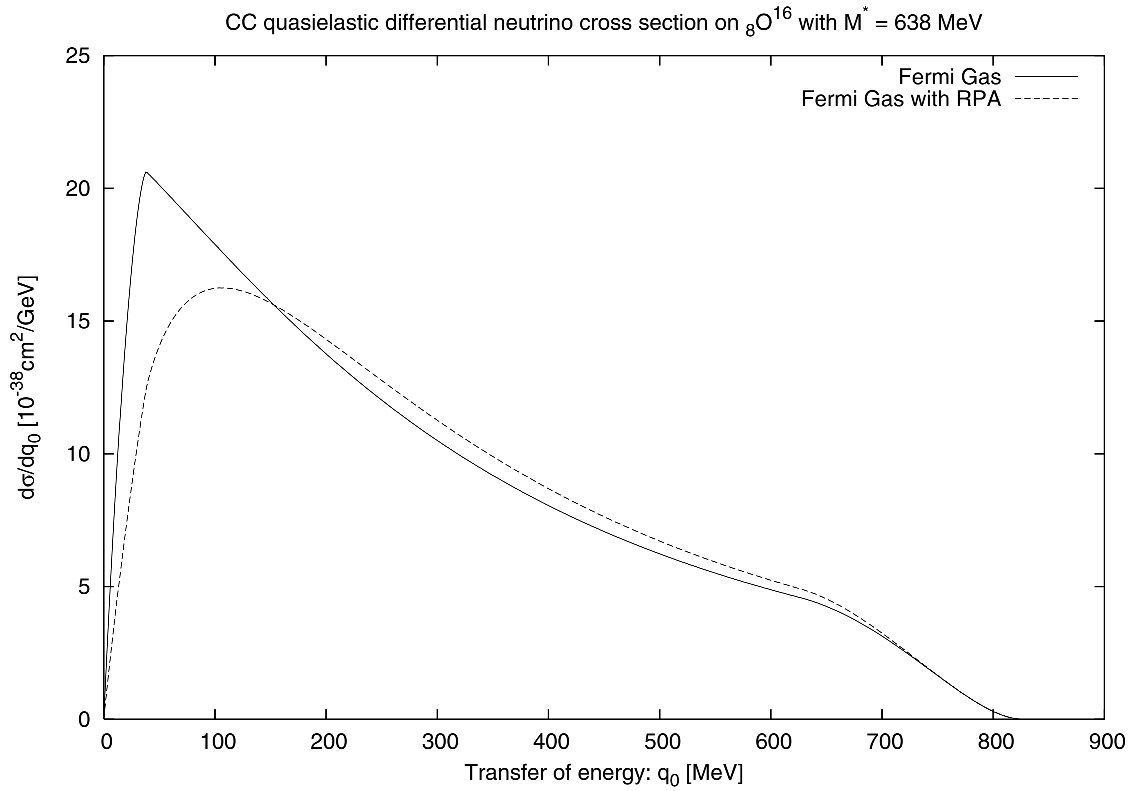

Fig. 11. Comparison of differential quasi-elastic neutrino cross sections for a Fermi gas of free nucleons with cross section modified by RPA corrections for a neutrino energy of 1 GeV. Calculations were done with effective mass $M^* = 638\,\mathrm{MeV}$ and $g' = 0.7$

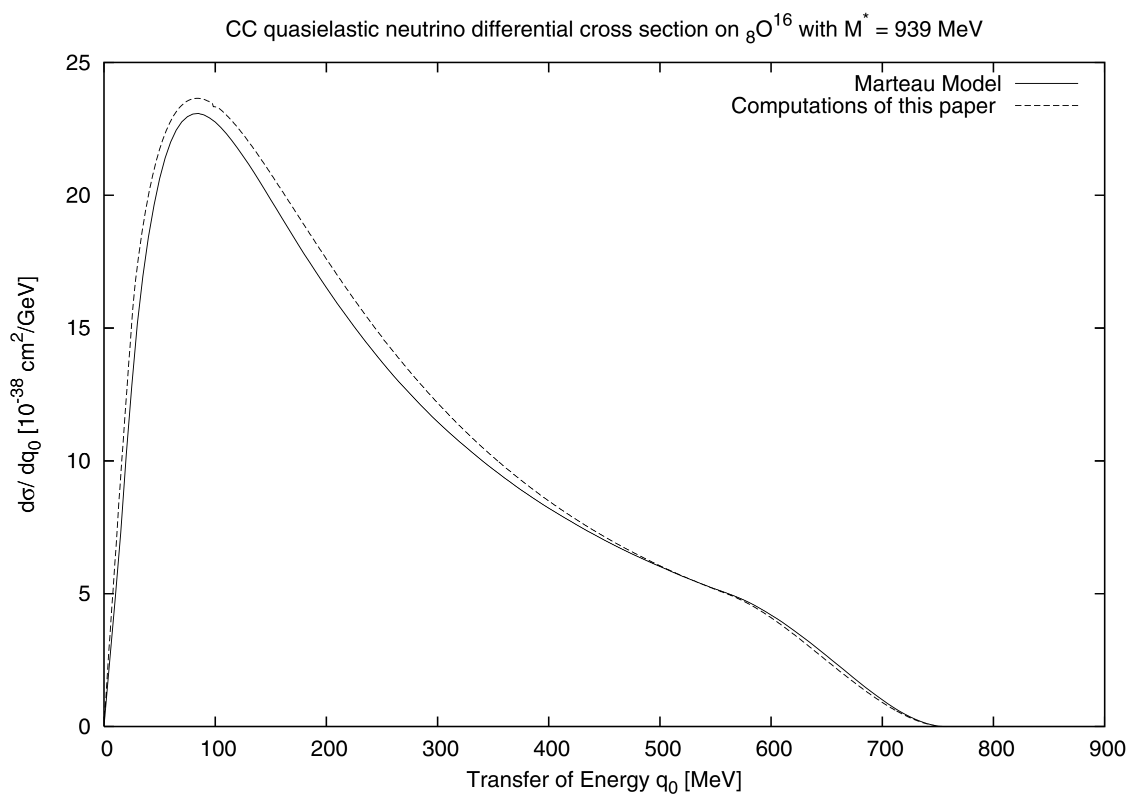

Fig. 12. Comparison of differential quasi-elastic neutrino cross sections with RPA corrections in the model of this paper and the Marteau model [12] for a neutrino energy of 1 GeV. The calculation was done for the free nucleon mass and $g' = 0.6$

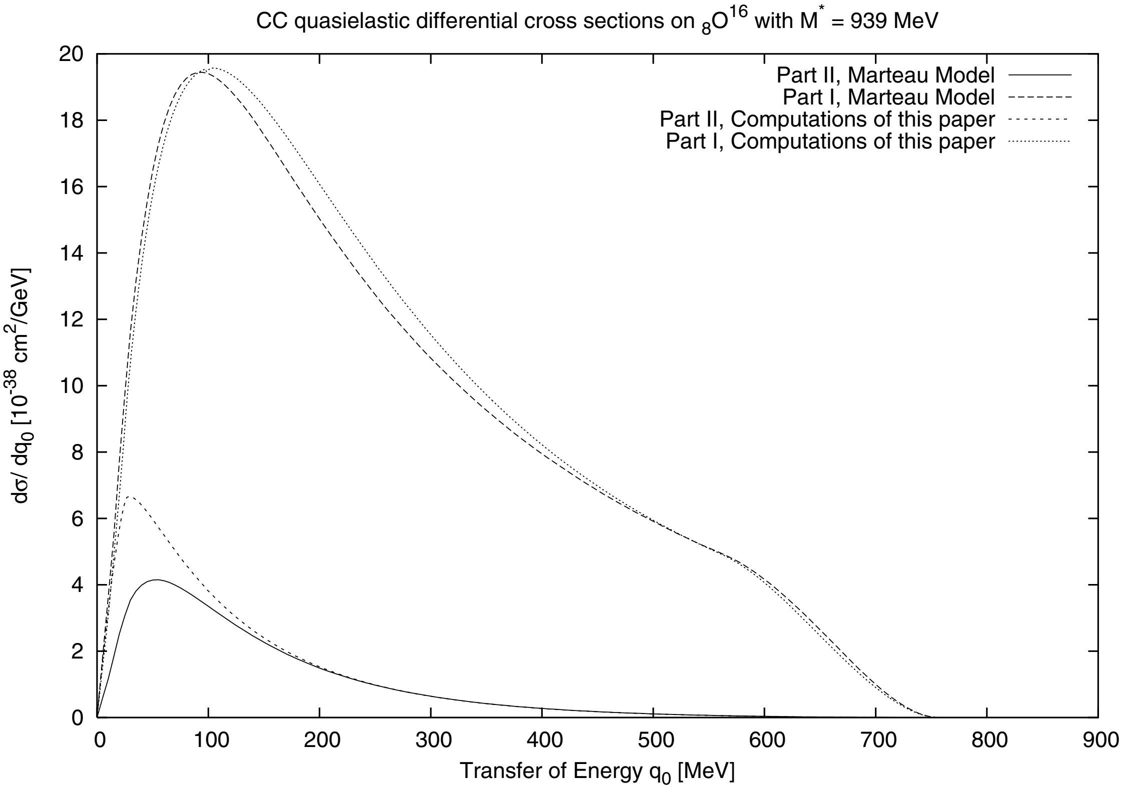

Fig. 13. Comparison of RPA corrections in contributions I and II, in the model of this paper and the Marteau model [12]. The calculation was done for the free nucleon mass and $g' = 0.6$

Appendix A

Equations (25) are rewritten as sixteen ones:

$$
\begin{aligned}
\Delta_1^{\mathrm{A}} &= W^{\mathrm{A}}, \\
\Delta_2^{\mathrm{A}} &= 0, \\
\Delta_3^{\mathrm{A}} &= V^{\mathrm{A}} \Delta_3^{\mathrm{A}} \Pi_{\pi\pi}^{\mathrm{A}}, \\
\Delta_4^{\mathrm{A}} &= V^{\mathrm{A}} \left(1 + \Delta_4^{\mathrm{A}} \Pi_{\pi\pi}^{\mathrm{A}}\right),
\end{aligned} \tag{33}
$$

$$
\begin{aligned}
\Delta_1^{\mathrm{L}} &= R^{\mathrm{L}} + \left(\Delta_1^{\mathrm{A}} + \Delta_1^{\mathrm{L}}\right) \left(W^{\mathrm{A}} + W^{\mathrm{L}}\right) \Pi_{\rho\rho}^{\mathrm{L}}, \\
\Delta_2^{\mathrm{L}} &= \left(\Delta_2^{\mathrm{A}} + \Delta_2^{\mathrm{L}}\right) \left(W^{\mathrm{A}} + W^{\mathrm{L}}\right) \Pi_{\rho\rho}^{\mathrm{L}}, \\
\Delta_3^{\mathrm{L}} &= \Delta_3^{\mathrm{A}} \left(\Pi_{\pi\pi}^{\mathrm{A}} V^{\mathrm{L}} + \Pi_{\pi\pi}^{\mathrm{L}} V^{\mathrm{A}} + \Pi_{\pi\pi}^{\mathrm{L}} V^{\mathrm{L}}\right) \\
&\quad + \Delta_3^{\mathrm{L}} \left(V^{\mathrm{A}} + V^{\mathrm{L}}\right) \left(\Pi_{\pi\pi}^{\mathrm{A}} + \Pi_{\pi\pi}^{\mathrm{L}}\right), \\
\Delta_4^{\mathrm{L}} &= V^{\mathrm{L}} + \Delta_4^{\mathrm{A}} \left(\Pi_{\pi\pi}^{\mathrm{A}} V^{\mathrm{L}} + \Pi_{\pi\pi}^{\mathrm{L}} V^{\mathrm{A}} + \Pi_{\pi\pi}^{\mathrm{L}} V^{\mathrm{L}}\right) \\
&\quad + \Delta_4^{\mathrm{L}} \left(V^{\mathrm{A}} + V^{\mathrm{L}}\right) \left(\Pi_{\pi\pi}^{\mathrm{L}} + \Pi_{\pi\pi}^{\mathrm{A}}\right),
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
\Delta_1^{\mathrm{T}} &= W^{\mathrm{T}} + \left(\Delta_1^{\mathrm{A}} + \Delta_1^{\mathrm{T}}\right) \Pi_{\rho\rho}^{\mathrm{T}} \left(W^{\mathrm{A}} + W^{\mathrm{T}}\right) \\
&\quad + \Delta_3^{\mathrm{VA}} \Pi_{\rho\pi}^{\mathrm{VA}} \left(W^{\mathrm{A}} + W^{\mathrm{T}}\right), \\
\Delta_2^{\mathrm{T}} &= \left(\Delta_2^{\mathrm{A}} + \Delta_2^{\mathrm{T}}\right) \Pi_{\rho\rho}^{\mathrm{T}} \left(W^{\mathrm{A}} + W^{\mathrm{T}}\right) \\
&\quad + \Delta_4^{\mathrm{VA}} \Pi_{\rho\pi}^{\mathrm{VA}} \left(W^{\mathrm{A}} + W^{\mathrm{T}}\right), \\
\Delta_3^{\mathrm{T}} &= \Delta_1^{\mathrm{VA}} \Pi_{\rho\pi}^{\mathrm{VA}} \left(V^{\mathrm{A}} + V^{\mathrm{T}}\right) \\
&\quad + \Delta_3^{\mathrm{T}} \left(\Pi_{\pi\pi}^{\mathrm{A}} + \Pi_{\pi\pi}^{\mathrm{T}}\right) \left(V^{\mathrm{A}} + V^{\mathrm{T}}\right) \\
&\quad + \Delta_3^{\mathrm{A}} \left(\Pi_{\pi\pi}^{\mathrm{A}} V^{\mathrm{T}} + \Pi_{\pi\pi}^{\mathrm{T}} V^{\mathrm{A}} + \Pi_{\pi\pi}^{\mathrm{T}} V^{\mathrm{T}}\right), \\
\Delta_4^{\mathrm{T}} &= V^{\mathrm{T}} + \Delta_2^{\mathrm{VA}} \Pi_{\rho\pi}^{\mathrm{VA}} \left(V^{\mathrm{A}} + V^{\mathrm{T}}\right) \\
&\quad + \Delta_4^{\mathrm{T}} \left(\Pi_{\pi\pi}^{\mathrm{T}} + \Pi_{\pi\pi}^{\mathrm{A}}\right) \left(V^{\mathrm{A}} + V^{\mathrm{T}}\right) \\
&\quad + \Delta_4^{\mathrm{A}} \left(\Pi_{\pi\pi}^{\mathrm{A}} V^{\mathrm{T}} + \Pi_{\pi\pi}^{\mathrm{T}} V^{\mathrm{A}} + \Pi_{\pi\pi}^{\mathrm{T}} V^{\mathrm{T}}\right),
\end{aligned} \tag{35}
$$

$$
\begin{aligned}
\Delta_1^{\mathrm{VA}} &= \left(\Delta_1^{\mathrm{VA}} \Pi_{\rho\rho}^{\mathrm{T}} + \Delta_3^{\mathrm{A}} \Pi_{\rho\pi}^{\mathrm{VA}} + \Delta_3^{\mathrm{T}} \Pi_{\rho\pi}^{\mathrm{VA}}\right) \left(W^{\mathrm{A}} + W^{\mathrm{T}}\right), \\
\Delta_2^{\mathrm{VA}} &= \left(\Delta_2^{\mathrm{VA}} \Pi_{\rho\rho}^{\mathrm{T}} + \Delta_4^{\mathrm{A}} \Pi_{\rho\pi}^{\mathrm{VA}} + \Delta_4^{\mathrm{T}} \Pi_{\rho\pi}^{\mathrm{VA}}\right) \left(W^{\mathrm{A}} + W^{\mathrm{T}}\right), \\
\Delta_3^{\mathrm{VA}} &= \left(\Delta_1^{\mathrm{A}} \Pi_{\rho\pi}^{\mathrm{VA}} + \Delta_1^{\mathrm{T}} \Pi_{\rho\pi}^{\mathrm{VA}} + \Delta_3^{\mathrm{VA}} \Pi_{\pi\pi}^{\mathrm{A}} + \Delta_3^{\mathrm{VA}} \Pi_{\pi\pi}^{\mathrm{T}}\right) \\
&\quad \times \left(V^{\mathrm{A}} + V^{\mathrm{T}}\right), \\
\Delta_4^{\mathrm{VA}} &= \left((\Delta_2^{\mathrm{A}} + \Delta_2^{\mathrm{T}}) \Pi_{\rho\pi}^{\mathrm{VA}} + \Delta_4^{\mathrm{VA}} \Pi_{\pi\pi}^{\mathrm{A}} + \Delta_4^{\mathrm{VA}} \Pi_{\pi\pi}^{\mathrm{T}}\right) \\
&\quad \times \left(V^{\mathrm{A}} + V^{\mathrm{T}}\right).
\end{aligned} \tag{36}
$$

We solve these equations sector-by-sector. Equations (33) clearly lead to

$$
\begin{aligned}
\Delta_1^{\mathrm{A}} &= W^{\mathrm{A}}, \\
\Delta_2^{\mathrm{A}} &= 0, \\
\Delta_3^{\mathrm{A}} &= 0, \\
\Delta_4^{\mathrm{A}} &= \frac{V^{\mathrm{A}}}{1 - V^{\mathrm{A}} \Pi_{\pi\pi}^{\mathrm{A}}}.
\end{aligned} \tag{37}
$$

To proceed it is convenient to define

$$
\begin{aligned}
R^{\mathrm{TA}} &= W^{\mathrm{T}} + W^{\mathrm{A}},\\
V^{\mathrm{TA}} &= V^{\mathrm{T}} + V^{\mathrm{A}},\\
V^{\mathrm{LA}} &= V^{\mathrm{L}} + V^{\mathrm{A}}.
\end{aligned}
\tag{38}
$$

Equations (34) contain only Δ_j^{A} and Δ_j^{L} components. We obtain

$$
\begin{aligned}
\Delta_1^{\mathrm{L}} &= \frac{W^{\mathrm{L}} + R^{\mathrm{LA}}\Pi_{\rho\rho}^{\mathrm{L}}W^{\mathrm{A}}}{1 - R^{\mathrm{LA}}\Pi_{\rho\rho}^{\mathrm{L}}},\\
\Delta_2^{\mathrm{L}} &= 0,\\
\Delta_3^{\mathrm{L}} &= 0,\\
\Delta_4^{\mathrm{L}} &= \frac{V^{\mathrm{L}} + V^{\mathrm{LA}}\Pi_{\pi\pi}^{\mathrm{L}}V^{\mathrm{A}}}{(1 - V^{\mathrm{A}}\Pi_{\pi\pi}^{\mathrm{A}})\,(1 - V^{\mathrm{LA}}(\Pi_{\pi\pi}^{\mathrm{L}} + \Pi_{\pi\pi}^{\mathrm{A}}))}.
\end{aligned}
\tag{39}
$$

Δ^{T} and Δ^{VA} components mix among themselves but always in pairs:

$$
\begin{aligned}
\Delta_1^{\mathrm{T}} &\longleftrightarrow \Delta_3^{\mathrm{VA}},\\
\Delta_2^{\mathrm{T}} &\longleftrightarrow \Delta_4^{\mathrm{VA}},\\
\Delta_3^{\mathrm{T}} &\longleftrightarrow \Delta_1^{\mathrm{VA}},\\
\Delta_4^{\mathrm{T}} &\longleftrightarrow \Delta_2^{\mathrm{VA}}.
\end{aligned}
\tag{40}
$$

We derive

$$
\begin{aligned}
&\Delta_1^{\mathrm{T}} \qquad (41)\\
&= \frac{\left[1 - V^{\mathrm{TA}}(\Pi_{\pi\pi}^{\mathrm{A}} + \Pi_{\pi\pi}^{\mathrm{T}})\right]\left[W^{\mathrm{T}} + W^{\mathrm{A}}W^{\mathrm{TA}}\Pi_{\rho\rho}^{\mathrm{T}}\right]}{\left[1 - V^{\mathrm{TA}}(\Pi_{\pi\pi}^{\mathrm{A}} + \Pi_{\pi\pi}^{\mathrm{T}})\right]\left[1 - R^{\mathrm{TA}}\Pi_{\rho\rho}^{\mathrm{T}}\right] - R^{\mathrm{TA}}V^{\mathrm{TA}}(\Pi_{\rho\pi}^{\mathrm{VA}})^2}\\
&+ \frac{W^{\mathrm{A}}R^{\mathrm{TA}}V^{\mathrm{TA}}(\Pi_{\rho\pi}^{\mathrm{VA}})^2}{\left[1 - V^{\mathrm{TA}}(\Pi_{\pi\pi}^{\mathrm{A}} + \Pi_{\pi\pi}^{\mathrm{T}})\right]\left[1 - R^{\mathrm{TA}}\Pi_{\rho\rho}^{\mathrm{T}}\right] - R^{\mathrm{TA}}V^{\mathrm{TA}}(\Pi_{\rho\pi}^{\mathrm{VA}})^2},\\
&\Delta_2^{\mathrm{T}} = 0,\\
&\Delta_3^{\mathrm{T}} = 0,\\
&\Delta_4^{\mathrm{T}} \qquad (42)\\
&= \frac{(1 - R^{\mathrm{TA}}\Pi_{\rho\rho}^{\mathrm{T}})\left(V^{\mathrm{T}} + \Delta_4^{\mathrm{A}}(V^{\mathrm{TA}}\Pi_{\pi\pi}^{\mathrm{T}} + V^{\mathrm{T}}\Pi_{\pi\pi}^{\mathrm{A}})\right)}{\left[1 - V^{\mathrm{TA}}\,(\Pi_{\pi\pi}^{\mathrm{A}} + \Pi_{\pi\pi}^{\mathrm{T}})\right]\left[1 - R^{\mathrm{TA}}\Pi_{\rho\rho}^{\mathrm{T}}\right] - V^{\mathrm{TA}}R^{\mathrm{TA}}(\Pi_{\rho\pi}^{\mathrm{VA}})^2}\\
&+ \frac{\Delta_4^{\mathrm{A}}V^{\mathrm{TA}}R^{\mathrm{TA}}(\Pi_{\rho\pi}^{\mathrm{VA}})^2}{\left[1 - V^{\mathrm{TA}}\,(\Pi_{\pi\pi}^{\mathrm{A}} + \Pi_{\pi\pi}^{\mathrm{T}})\right]\left[1 - R^{\mathrm{TA}}\Pi_{\rho\rho}^{\mathrm{T}}\right] - V^{\mathrm{TA}}R^{\mathrm{TA}}(\Pi_{\rho\pi}^{\mathrm{VA}})^2},\\
&\Delta_1^{\mathrm{VA}} = 0,\\
&\Delta_2^{\mathrm{VA}} \qquad (43)\\
&= \frac{W^{\mathrm{AT}}\Pi_{\rho\pi}^{\mathrm{VA}}(V^{\mathrm{T}} + (1 - \Pi_{\pi\pi}V^{\mathrm{A}})\Delta_4^{\mathrm{A}})}{1 - W^{\mathrm{AT}}(\Pi_{\rho\rho}^{\mathrm{T}} + V^{\mathrm{AT}}(\Pi_{\rho\pi}^{\mathrm{VA}})^2) + V^{\mathrm{AT}}(\Pi_{\pi\pi}^{\mathrm{A}} + \Pi_{\pi\pi}^{\mathrm{T}})(\Pi_{\rho\rho}^{\mathrm{T}}W^{\mathrm{AT}} - 1)},\\
&\Delta_3^{\mathrm{VA}} \qquad (44)\\
&= \frac{V^{\mathrm{AT}}W^{\mathrm{AT}}\Pi_{\rho\pi}^{\mathrm{VA}}}{1 - W^{\mathrm{AT}}(\Pi_{\rho\rho}^{\mathrm{T}} + V^{\mathrm{AT}}(\Pi_{\rho\pi}^{\mathrm{VA}})^2) + V^{\mathrm{AT}}(\Pi_{\pi\pi}^{\mathrm{A}} + \Pi_{\pi\pi}^{\mathrm{T}})(\Pi_{\rho\rho}^{\mathrm{T}}W^{\mathrm{AT}} - 1)},\\
&\Delta_4^{\mathrm{VA}} = 0.
\end{aligned}
$$

Acknowledgements. This work was supported by KBN grant 105/E-344/SPB/ICARUS/P-03/DZ 211/2003-2005. The authors thank the anonymous referee for questions, remarks and suggestions which led to substantial improvement of the paper.

References

1. A. Para, Nucl. Phys. B (Proc. Suppl.) **112**, 9 (2002); P. Lipari, Nucl. Phys. B (Proc. Suppl.) **112**, 274 (2002)
2. B.D. Serot, J.D. Walecka, Advances in Nuclear Physics, edited by J.W. Negele, E. Vogt (Plenum, New York 1986), vol. 16
3. R.A. Smith, E.J. Moniz, Nucl. Phys. B **43**, 605 (1972)
4. F. Osterfeld, Rev. Mod. Phys. **64**, 491 (1992)
5. A.L. Fetter, J.D. Walecka, Quantum theory of many-particle system (McGraw-Hill, New York 1971)
6. Z.A. Meziani et al., Phys. Rev. Lett. **54**, 1233 (1985)
7. J. Marteau, Eur. J. Phys. A **5**, 183 (1999); De l'effet des interactions nucléaires dans les reactions de neutrinos sur des cibles d'oxygene et de son role dans l'anomalie des neutrinos atmospheriques, Ph.D. Thesis, supervisor J. Delorme, Lyon 1999
8. J. Marteau, private communication
9. S.K. Singh, E. Oset, Nucl. Phys. A **542**, 587 (1992); Phys. Rev. C **48**, 1246 (1993); J. Engel, E. Kolbe, K. Langancke, P. Vogel, Phys. Rev. D **48**, 3048 (1993)
10. H. Kim, J. Piekarewicz, C.J. Horowitz, Phys. Rev. C **51**, 2739 (1995)
11. L. Mornas, A. Perez, Eur. Phys. J. A **13**, 383 (2002); L. Mornas, nucl-th/0210035
12. J.T. Sobczyk, Modelling nuclear effects in neutrino interactions in 1 GeV region, nucl-th/0307047
13. L. Llewellyn Smith, Phys. Rep. **3**, 261 (1972)
14. J.D. Walecka, Semileptonic weak interactions in nuclei, Proceeding, Weak Interactions Physics (New York 1977), pp. 125–147
15. S. Chin, Ann. Phys. **108**, 301 (1977); C.J. Horowitz, Nucl. Phys. A **412**, 228 (1984); C.J. Horowitz, K. Wehrberger, Nucl. Phys. A **531**, 665 (1991)